A GRAPHICAL METHOD FOR THE DESIGN OF STEPPED IMPEDANCE TRANSFORMERS*

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A graphical solution using families of generalized curves, for designing a stepped impedance transformer is presented. The error for a numerical example is less than 2.2% when number of section is seven and the accuracy improves as the number of sections increases.

Optimum designs of stepped impedance transformers (SIT) have been frequently reported by many authors. Exact solutions [1,5], approximate solutions[2] as well as design tables[4,5] are well covered in the literature. In this paper a graphical design procedure of SIT with equal-ripple response is presented. The merits of a graphical design are three-fold: (a) it may be helpful to some engineers who do not like the complicated analytical equations when the available design tables are not sufficient for certain design requirements; (b) a value obtained graphically may serve as a starting point in a computer-aided design[6] to reduce the computational time when high accuracy is needed; (c) the solution is simple and is applicable to large number of sections and any ratio of impedances.

Consider the n-section SIT as shown in Fig. 1 where Z_k, k = 1,2,3, ... n is the characteristic impedance of the k-th section normalized with respect to the characteristic impedance of the input transmission line, and where the normalized load resistance is R. It can be shown that

$$\frac{Z_k}{Z_{k+1}} = \frac{Z_{n-k}}{Z_{n-k+1}} \qquad (1)$$

for all values of k. For convenience, the following parameters are introduced:

$$x_k = \frac{2k}{n+1} - 1 \qquad (2)$$

$$y_k = 2(\log Z_k/\log R) - 1 \qquad (3)$$

and

$$\beta = \sqrt{\frac{S_{max}}{R}}\ \frac{R-1}{S_{max}-1} \qquad (4)$$

where S_{max} is the maximum voltage standing wave ratio allowed within the bandwidth $(f_2 - f_1)$, f_2 and f_1 are the upper and lower frequencies in the frequency band, respectively. The number of sections of SIT, n, is related to the ratio f_2:f_1, R, and S_{max} by [5]

$$\beta = \cosh\left[n \cosh^{-1}\left(\sec \frac{\pi}{1+f_2/f_1}\right)\right] \qquad (5)$$

In general, for given values of R, S_{max} and f_2:f_1, the numerical value of n determined by (5) is not an integer. In this case, the next larger integer should be chosen.

From the numerical observations of the exact solutions[5†], it is found that the curves of x_k vs. y_k depend on the value of β, but are insensitive with respect to the individual value of n or f_2:f_1, if n is larger than 10. Using this empirical conclusion, a set of x_k vs. y_k curves with β as parameter, is plotted in Fig. 5 for n = 18. Actually, the error is less than 1 per cent if $n \geq 10$, and is less than 3 per cent if $n \geq 5$. It is noted that these curves are symmetrical with respect to the origin because of (1).

Since inaccuracy is inherited in graphical solutions, it may be assumed that Fig. 5 is valid for $n \geq 5$; but it should be kept in mind that more accurate results are achieved for larger values of n. With the curves in Fig. 5 being independent of n, the graphical method for designing an SIT is established by using families of curves which are obtained as follows:

(i) From (4), plot β vs. S_{max} with R as the parameter as depicted in Fig. 2;
(ii) From (5), a set of β vs. f_2:f_1 is plotted in Fig. 3 with n as the parameter;
(iii) Finally, on a semi-log graph, another set of straight lines of (2) can be easily drawn on a linear scale graph as shown in Fig. 4 where n is the parameter;
(iv) Finally, on a semi-log graph, another set of straight lines (Eq. (3)), Z_k vs. y_k with the parameters R is shown in Fig. 6.

Usually, the values of R, S_{max} and f_2:f_1 are given for designing an SIT. With these given requirements, one may use Fig. 2 to locate the proper value of β, then Fig. 3 to determine the number of sections n. If the value of n corresponding to the known β, and f_2:f_1 is not an integer,

† It is observed that the method outlined in [5] for numerical computation yields significant error when n is larger than 16. Modified computational procedures have been introduced to reduce the accumulative numerical error.

choose the next larger integer. For the determined n and β, value of x_k corresponding to each k, k = 1,2,3,.... n, is obtained from Fig. 4, then, y_k is given by Fig. 5. Finally, from Fig. 6, Z_k is determined by R and the corresponding value of y_k for each k. Observe that, except Fig. 5, other graphs can be replaced by Eqs. (2) through (5) correspondingly.

In order to assess this graphical method, consider a design of SIT with R = 4, S_{max} = 1.128 and f_2:f_1 = 6.08 as an example. First, from Figs. 2 and 3 (or (4) and (5)), β = 12.4 and n = 7 are determined. Next, from Fig. 4 (or (2)), Fig. 5, and Fig. 6 (or (3)), the characteristic impedances of the seven sections are obtained as listed in Table I. The exact values of Z_k are also listed for comparison, they are obtained from the table given in [5] with

TABLE I

k	1	2	3	4	5	6	7
x_k	-0.75	-0.5	-0.25	0	0.25	0.5	0.75
y_k	-0.8	-0.58	-0.32	0	0.35	0.58	0.8
Z_{kg}	1.13	1.32	1.6	2.0	2.5	3.0	3.5
Z_{ke}	1.126	1.305	1.592	2.0	2.513	3.065	3.551

k = 0.6, ρ_{max} = 0.06, BW = 1.435. The maximum error of the graphical solutions is less than 2.2%. The error is smaller if n is larger. There is no essential limit on the value of n.

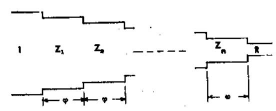

Fig. 1 Stepped Impedance Transformer

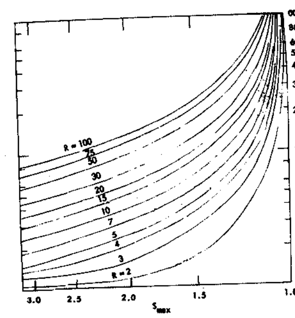

Figure 2

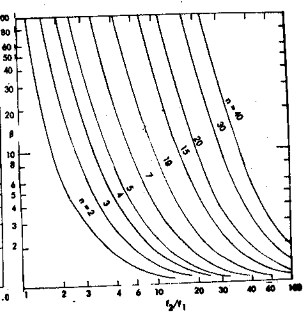

Figure 3

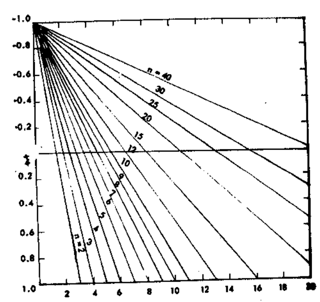

Figure 4

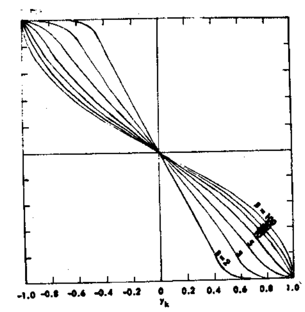

Figure 5

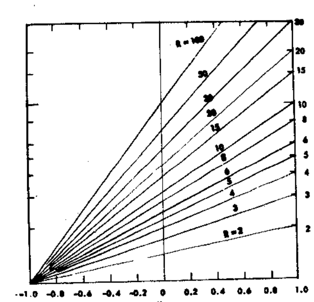

Figure 6

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